Subject : Mathematics

Teacher name : Archana Jain Semester 1st

Name of the Paper : Paper 3

Class: MSc (Mathematics)

January2021

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| Section-I (Two Questions) Definition and examples of topological spaces, Neighbourhoods, Neighbourhood system of a point and its properties, Interior point and interior of a set, interior as an operator and its properties, definition of a closed set as complement of an open set, limit point (accumulation point) of a set, derived set of a set, definition of closure of a set as union of the set and its derived set, Adherent point (Closure point) of a set , closure of a set as set of adherent (closure) points, properties of closure, closure as an operator and its properties, boundary of a set, Dense sets. A characterization of dense sets. Base for a topology and its characterization, Base for Neighbourhood system, Sub-base for a topology. Relative (induced) Topology and subspace of a topological space. Alternate methods of defining a topology using ‘properties’ of ‘Neighbourhood system’, ‘Interior Operator’, ‘Closed sets’, Kuratowski closure operator and ‘base’. First countable, Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and a second countable space, Lindelof theorem. Comparison of Topologies on a set, about intersection and union of topologies, infimum and supremum of a collection of topologies on a set, the collection of all topologies on a set as a complete lattice . |

February 2021

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| Definition, examples and characterisations of continuous functions, composition of continuous functions, Open and closed functions, Homeomorphism, embedding. Tychonoff product topology in terms of standard (defining) subbase, projection maps, their continuity and openness, Characterisation of product topology as the smallest topology with projections continuous, continuity of a function from a space into a product of spaces. T0 , T1 , T2 ,Regular and T3 separation axioms, their characterization and basic properties i.e. hereditary property of T0 , T1 , T2 , Regular and T3 spaces, and productive property of T1 and T2 spaces. Quotient topology w.r.t. a map, Continuity of function with domain a space having quotient topology, About Hausdorffness of quotient space Completely regular and Tychonoff (T 3 1/2), spaces, their hereditary and productive properties. Embedding lemma, Embedding theorem. Normal and T4 spaces : Definition and simple examples, Urysohn’s Lemma, complete regularity of a regular normal space, T4 implies Tychonoff, Tietze’s extension theorem (Statement only). Definition and examples of filters on a set, Collection of all filters on a set as a p.o. set, finer filter, methods of generating filters/finer filters, Ultra filter (u.f.) and its characterizations, Ultra Filter Principle (UFP) i.e. Every filter is contained in an ultra filter. Image of filter under a function. Convergence of filters: Limit point (Cluster point) and limit of a filter and relationship between them, Continuity in terms of convergence of filters. Hausdorffness and filter convergence. |

March 2021

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| Compactness: Definition and examples of compact spaces, definition of a compact subset as a compact subspace, relation of open cover of a subset of a topological space in the sub-space with that in the main space, compactness in terms of finite intersection property (f.i.p.), continuity and compact sets, compactness and separation properties, Closedness of compact subset, closeness of continuous map from a compact space into a Hausdorff space and its consequence, Regularity and normality of a compact Hausdorff space. Compactness and filter convergence, Convergence of filters in a product space, compactness and product space. Tychonoff product theorem using filters, Tychonoff space as a subspace of a compact Hausdorff space and its converse, compactification and Hausdorff compactification, Stone-Cech compactification |